LESSON 4.1b

Graphing Polynomial Functions

Today you will:

- Graph polynomial functions using tables and end behavior
- Practice using English to describe math processes and equations

Core Vocabulary:

- polynomial, p. 158
- polynomial function, p. 158
- end behavior, p. 159

Previous:

- monomial
- linear function
- quadratic function

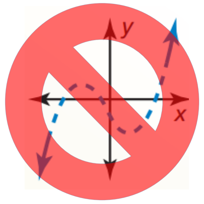
Graph (a) $f(x) = -x^3 + x^2 + 3x - 3$ and (b) $f(x) = x^4 - x^3 - 4x^2 + 4$.

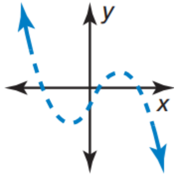
SOLUTION

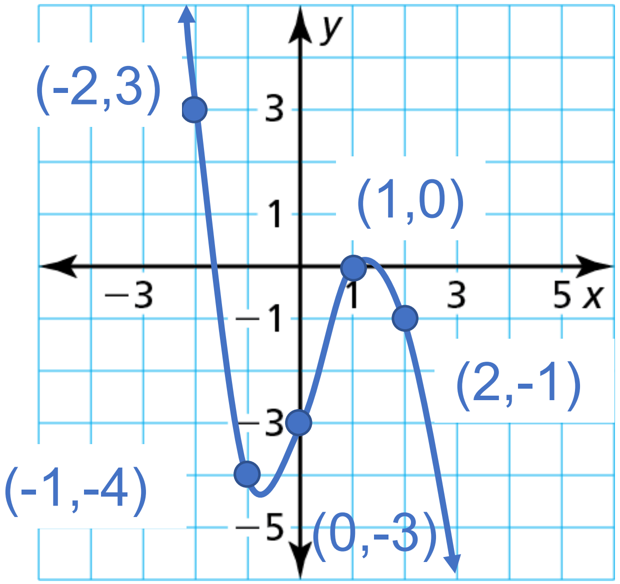
a. To graph the function, make a table of values and plot the corresponding points. Connect the points with a smooth curve and check the end behavior.

x	-2	-1	0	1	2
$f(x)$	3	-4	-3	0	-1

The degree is odd
and the leading coefficient is negative.
So, $f(x) \to +\infty$ as $x \to -\infty$ and $f(x) \to -\infty$ as $x \to +\infty$.



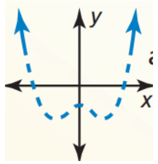


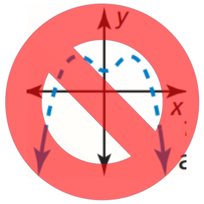


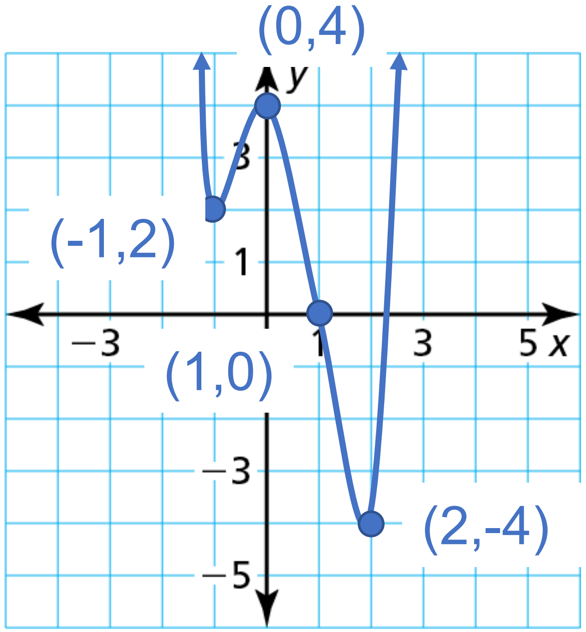
b. To graph the function, make a table of values and plot the corresponding points. Connect the points with a smooth curve and check the end behavior.

x	-2	-1	0	1	2
$f(x)$	12	2	4	0	-4

The degree is even
and the leading coefficient is positive.
So, $f(x) \to +\infty$ as $x \to -\infty$ and $f(x) \to +\infty$ as $x \to +\infty$.





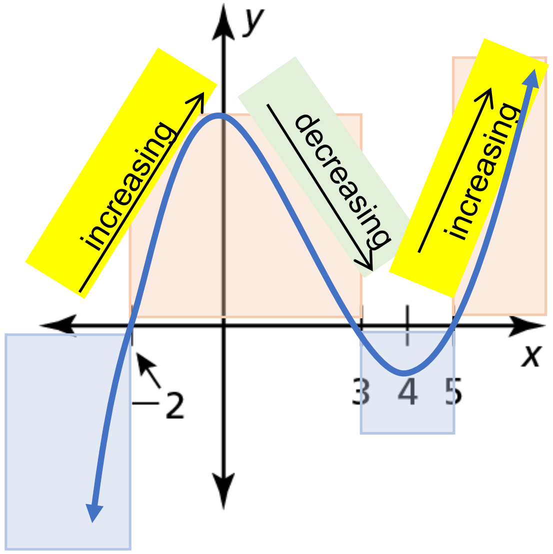


Sketch a graph of the polynomial function *f* having these characteristics.

- *f* is increasing when $x < 0$ and $x > 4$.
- *f* is decreasing when $0 < x < 4$.
- $f(x) > 0$ when $-2 < x < 3$ and $x > 5$.
- $f(x) < 0$ when $x < -2$ and $3 < x < 5$.

Use the graph to describe the degree and leading coefficient of *f*.

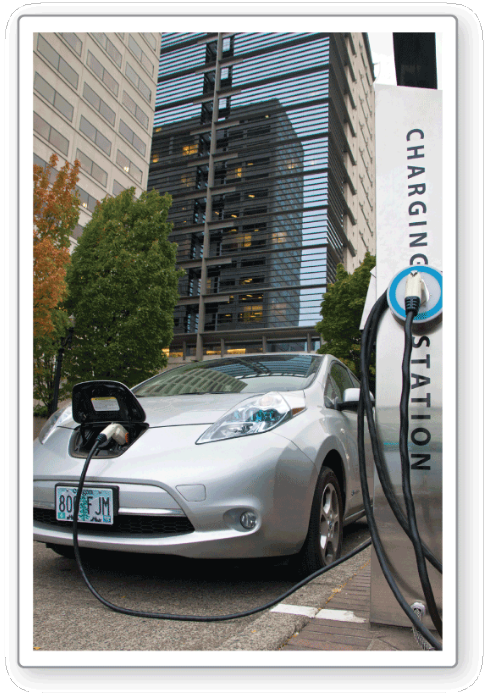
SOLUTION



The graph is above the *x*-axis when $f(x) > 0$.

The graph is below the *x*-axis when $f(x) < 0$.

From the graph, $f(x) \to -\infty$ as $x \to -\infty$ and $f(x) \to +\infty$ as $x \to +\infty$.
So, the degree is odd and the leading coefficient is positive.



The estimated number V (in thousands) of electric vehicles in use in the United States can be modeled by the polynomial function

$$V(t) = 0.151280t^3 - 3.28234t^2 + 23.7565t - 2.041$$

where t represents the year, with $t = 1$ corresponding to 2001.

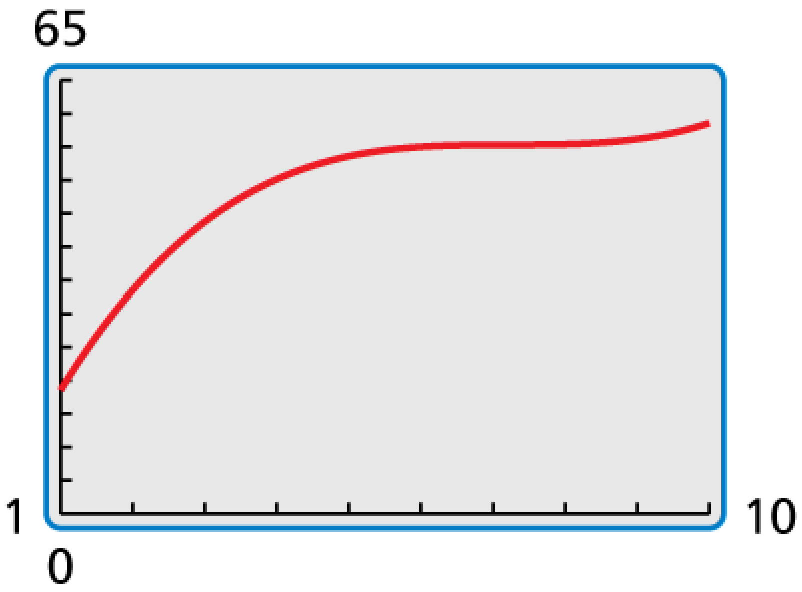
a. Use a graphing calculator to graph the function for the interval $1 \leq t \leq 10$. Describe the behavior of the graph on this interval.

b. What was the average rate of change in the number of electric vehicles in use from 2001 to 2010?

c. Do you think this model can be used for years before 2001 or after 2010? Explain your reasoning.

SOLUTION

a. Using a graphing calculator and a viewing window of $1 \leq x \leq 10$ and $0 \leq y \leq 65$, you obtain the graph shown.



From 2001 to 2004, the numbers of electric vehicles in use increased. Around 2005, the growth in the numbers in use slowed and started to level off. Then the numbers in use started to increase again in 2009 and 2010.

From 2001 to 2004, the numbers of electric vehicles in use increased. Around 2005, the growth in the numbers in use slowed and started to level off. Then the numbers in use started to increase again in 2009 and 2010.

b. The years 2001 and 2010 correspond to $t = 1$ and $t = 10$.

Average rate of change over $1 \le t \le 10$:

$$\frac{V(10) - V(1)}{10 - 1} = \frac{58.57 - 18.58444}{9} \approx 4.443$$

The average rate of change from 2001 to 2010 is about 4.4 thousand electric vehicles per year.

c. Because the degree is odd and the leading coefficient is positive, $V(t) \to -\infty$ as $t \to -\infty$ and $V(t) \to +\infty$ as $t \to +\infty$. The end behavior indicates that the model has unlimited growth as t increases. While the model may be valid for a few years after 2010, in the long run, unlimited growth is not reasonable. Notice in 2000 that $V(0) = -2.041$. Because negative values of $V(t)$ do not make sense given the context (electric vehicles in use), the model should not be used for years before 2001.

Homework

Pg 162, #23-43